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MODELING THE TURBULENT TRANSPORT OF AN IMPULSE IN THE WAKE OF A CYLINDER WITH THE USE OF EQUATIONS FOR THIRD MOMENTS

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1. One trend in modern phenomenological theory of turbulent transport is the formulation of a system of equations for the moments of the hydrodynamic fields of a turbulent flow, the maximum order of which is usually predicted with the aid of both physical considerations and the chosen method of closing the system. Models of turbulent transport have recently been proposed that are closed at the level of second moments — in which the unknowns are second moments — and in which third moments are modeled on the basis of heuristic considerations. Equations for moments of higher order are ultimately attractive for the reason that, in a whole range of physical problems, the turbulent transport of impulses, heat, or scalar properties cannot be correctly described within the framework of the simplest first-order gradient models (such as the Prandtl theory of displacement paths). Such problems are not the exception, and several of them may be found in [1-4]. An example of a model of turbulent transport closed at the level of the second moments (second-order model) would be the model [5] in which the turbulent flows (i.e., the second moments of turbulent fluctuations) are closed by means of the use of the method of the kinetic theory of gases in connection with third moments. Here, in essence a rough analogy is being made with kinetic theory, with the following justification: if a rough approximation for second-order moments makes it possible to compute first-order values in the simplest cases (this is true with the simplest phenomenological models of turbulent transport, based on the length of displacement paths), then it is possible that similar coarse approximations will make it possible to correctly predict second

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moments. Despite the fact that the use of second-order models has made it possible to correctly describe turbulent transport in problems for which the simplest phenomenological models of transport have proved inadequate, all second-order models have a basic shortcoming: they do not provide an efficient method of computing third moments.

The results obtained thus far in the numerical modeling of turbulent transport by means of second-order transport models for several specific physical problems (see, e.g., [4-8]) provide a basis for concluding that many phenomena of turbulent transport in a developed (three-dimensional) turbulence may be correctly described if third moments describing the processes of turbulent diffusion of this or that second moment of a hydrodynamic field (velocity, concentration, temperature, salinity, etc.) are included as unknowns in the turbulent transport model in a physically consistent manner. This would make it possible to surmount the obstacles connected with gradient models of turbulent transport in problems in which the temporal and spatial scales of turbulent movement are not small in relation to the temporal and spatial scales of the average movement (gradient models of transport assume that the turbulent scales are small compared to the scales of the average movement). One of the many examples of the unfitness of the gradient transport mechanism is the surface layer of the earth's atmosphere, mixed by turbulent convection (buoyancy forces). In this case, a gradient model of turbulent transport would not even be qualitatively correct in predicting either the distribution of divergences of the turbulent energy flow across this layer or the distribution of the flow itself — these are distributions that are actually observed in the atmosphere (see, e.g., [2-4]). It may be noted that there is presently a lively discussion going on relative to the question of the fitness of the gradient mechanism of turbulent transport [1, 9, 10].

Flow in the wake of a cylinder at large Reynolds numbers is also a case where the gradient mechanism of transport "doesn't work," at least in part of this flow [11]. According to the empirical data of Townsend [12], in the far region of the wake, where the statistical characteristics of the velocity field of the wake (second and third moments) are in an approximately similar condition, close to the axis of the wake there is a small region in which the density of the transverse component of a turbulent flow of longitudinal intensity $\langle u'^2\rangle$, i.e., the quantity $\langle u'^2 v'\rangle$ has the same sign as the gradient of this intensity $\partial\langle u'^2\rangle/\partial y$, so that close to the axis of the wake the longitudinal intensity will go in the direction of an increase in the gradient.

2. A model equation for the probability density of the velocity field proposed in [13, 14] has been written with allowance for pulsations of temperature and impurity concentration for an incompressible fluid (the equation and the assumptions on which it is based are explained in detail in [14]). The system of equations for the moments of the velocity field of a developed free turbulent flow, resulting from the transport equation derived from the above model equation for velocity field probability density, has the form

$$\frac{\partial\langle u_k\rangle}{\partial x_k}=0; \tag{2.1}$$

$$\frac{\partial\langle u_\alpha\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k\rangle\langle u_\alpha\rangle+\langle u_k'u_\alpha'\rangle\right]=0; \tag{2.2}$$

$$\begin{aligned}&\frac{\partial\langle u_\alpha'u_\beta'\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k\rangle\langle u_\alpha'u_\beta'\rangle+\langle u_k'u_\alpha'u_\beta'\rangle\right]+\langle u_k'u_\alpha'\rangle\frac{\partial\langle u_\beta\rangle}{\partial x_k}\\&+\langle u_k'u_\beta'\rangle\frac{\partial\langle u_\alpha\rangle}{\partial x_k}=-\frac{2\nu_1}{\tau}\langle u_\beta'u_\alpha'\rangle-\frac{\nu_2}{\tau}\left[\langle u_\alpha'u_\beta'\rangle-\frac{2}{3}E\delta_{\alpha\beta}\right];\end{aligned} \tag{2.3}$$

$$\begin{aligned}&\frac{\partial\langle u_\alpha'u_\beta'u_\gamma'\rangle}{\partial t}+\frac{\partial}{\partial x_k}\langle u_k\rangle\langle u_\alpha'u_\beta'u_\gamma'\rangle+\langle u_k'u_\beta'u_\gamma'\rangle\frac{\partial\langle u_\alpha\rangle}{\partial x_k}\\&+\langle u_k'u_\alpha'u_\gamma'\rangle\frac{\partial\langle u_\beta\rangle}{\partial x_k}+\langle u_k'u_\alpha'u_\beta'\rangle\frac{\partial\langle u_\gamma\rangle}{\partial x_k}-\langle u_\gamma'u_\beta'\rangle\frac{\partial}{\partial x_k}\langle u_\alpha'u_k'\rangle\\&-\langle u_\gamma'u_\alpha'\rangle\frac{\partial}{\partial x_k}\langle u_k'u_\beta'\rangle-\langle u_\alpha'u_\beta'\rangle\frac{\partial\langle u_\gamma'u_k'\rangle}{\partial x_k}+\frac{\partial}{\partial x_k}\langle u_k'u_\alpha'u_\beta'u_\gamma'\rangle=-\left(\frac{3\nu_2}{2}+3\nu_1\right)\frac{\langle u_\alpha'u_\beta'u_\gamma'\rangle}{\tau}.\end{aligned} \tag{2.4}$$

The angular brackets in Eqs. (2.1)-(2.4) are used to denote mean values, while the quote marks denote turbulent fluctuations of velocity field components. Relaxation time $\tau = AL/\sqrt{E}$, where $E = 1/2\langle u'_k u'_k\rangle$ is the density of the energy of the turbulence; L is the integral scale of the turbulence. Empirical constants A, ν_1, and ν_2 were determined from an examination of several particular cases [15] and do not change in the calculation of more complex flows [8] ($A \simeq 4$, $\nu_1 \simeq 1/2$, $\nu_2 \simeq 1$).

The system of equations for the moments (2.1)-(2.4) is not closed. Besides the fourth moment, appearing in the equation for the third moments (2.4), the scale of turbulence L remains as yet undetermined. To close system (2.1)-(2.4) (i.e., relate the fourth moments to the second) we used two approximations: Millionshchikov's hypothesis of quasinormality, and the method of thirteen moments of the kinetic theory of gases [16]. The former has been substantiated for the case of homogeneous turbulence [17]. It is evidently difficult to give strict proof that Millionshchikov's hypothesis can be correctly used for inhomogeneous turbulent flows, i.e., to guarantee that the third moments never exceed their physically possible values. However, it remains a fact that, given a physically correct approximation of the correlation between the pressure and velocity pulsation gradient, it is possible in different types of turbulent transport problems to obtain results in which nonphysical behavior of such different statistical characteristics as velocity field [5-8] and concentration field [8] is absent. To this may be added the fact that third moments differing from zero indicate a non-Gaussian distribution of probabilities, although, as noted in [4], if an expansion of perturbations about an equilibrium Gaussian distribution is used, it turns out that the relaxation time for third moments is about 20% of the relaxation time for second moments. Thus, if the relaxation time for each subsequent cumulant if less percentagewise, then it is justifiable to use the hypothesis of fourth moments if the deviation from the state of equilibrium is not too large. As is known, Millionshchikov's hypothesis is purely statistical in nature and is is no way connected with the specific mechanism of (turbulent) transport.

On the other hand, an analogy with molecular transport lies at the basis of the approach to the theory of turbulent transport founded on the use of equations to determine the common probability densities of the pulsations of the fields in question. It is useful to take this analogy somewhat further. Based on the empirical fact that the magnitude of the displacement path (or scale of correlation) is not small compared to the characteristic scale of the entire flow, an analogy should be made with molecular transport at large free paths. In this case, Fick's law for heat flow is already no longer valid, nor is Stokes' rheological equation of state for frictional stresses compared to the limiting laws at short free-path lengths. In the kinetic theory of gases, the transport equation for long free paths may be, e.g., the equation for stress tensors in the τ-approximation or thirteen-moments approximation [16]. Here, the stress tensor is the dependent variable, satisfying a first-order differential equation in partial derivatives. Using Grade's method, a closed system of equations of thirteen moments was obtained with a cut-off of the expansion of the distribution function into a series of Hermite polynomials at the third term, from which the relationship between the fourth and second moments follows. In the case of turbulent flow, this relationship takes the form

$$\begin{aligned}\langle u_\alpha' u_\beta' u_\gamma' u_k'\rangle &= h^2(\delta_{\alpha\beta}\delta_{k\gamma}+\delta_{\alpha k}\delta_{\beta\gamma}+\delta_{\alpha\gamma}\delta_{\beta k})\\ &+h\,\{\delta_{k\gamma}(\langle u_\alpha' u_\beta'\rangle-h\delta_{\alpha\beta})+\delta_{\beta\gamma}(\langle u_\alpha' u_k'\rangle-h\delta_{\alpha k})+\delta_{\beta k}(\langle u_\alpha' u_\gamma'\rangle-h\delta_{\alpha\gamma})\\ &+\delta_{\alpha\gamma}(\langle u_\beta' u_k'\rangle-h\delta_{\beta k})+\delta_{\alpha k}(\langle u_\beta' u_\gamma'\rangle-h\delta_{\beta\gamma})+\delta_{\alpha\beta}(\langle u_k' u_\gamma'\rangle-h\delta_{k\gamma})\},\end{aligned} \tag{2.5}$$

where $h = 2E/3$.

According to Millionshchikov's hypothesis, the fourth moments of the velocity field are expressed through the second by means of formulas that are valid for a normal distribution:

$$\langle u_\alpha' u_\beta' u_\gamma' u_k'\rangle = \langle u_\alpha' u_\beta'\rangle\langle u_\gamma' u_k'\rangle+\langle u_\alpha' u_\gamma'\rangle\langle u_\beta' u_k'\rangle+\langle u_\alpha' u_k'\rangle\langle u_\beta' u_\gamma'\rangle. \tag{2.6}$$

Either Eq. (2.5) or Eq. (2.6) closes (from the point of view of determining the scale of turbulence L) the system of equations for the velocity field moments (2.1)-(2.4).

All of the above concerning two possible methods of closing the system of equations for the moments of the velocity field of a developed, inhomogeneous turbulent flow provides a basis for proposing that it is of interest on its own account to compare both methods of closure in a problem where the turbulent flow is nongradient (nonderivative) in nature. The problem of flow in the wake of a cylinder is an appropriate and simple example (although it is not that simple as far as its mathematical realization is concerned).

Sequential development of a model of turbulent transport based on an equation for a single-point probability density (in the τ-approximation) also requires a sequential method

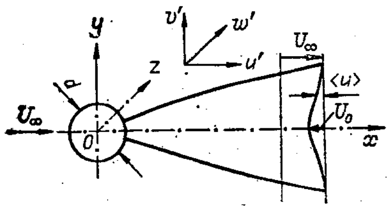

Fig. 1

of determining the scale of turbulence L within the framework of the model. For this scale to be interpreted as a correlation scale, within the limits of which an element of the fluid executes correlated pulsation in the surrounding medium, we have to examine an equation for a two-point density probability. This is one of the problems associated with the given transport model and which has yet to be fully resolved, i.e., no equation in realizable form has been derived for a two-point probability density. Thus, in solving the problem of flow in the wake of a cylinder (as with other similar problems of free turbulence), the expression for the scale of turbulence L may be chosen to have a form in accordance with a similar flow structure observed empirically. Only one scale magnitude is used.

3. System of equations (2.1)-(2.4), for the problem of flow in the wake of a cylinder (Fig. 1), may be written in an approximation of a boundary layer, while the equation of motion may be written in linearized form in the absence of a pressure gradient. These are conventional approximations in the problem of a flat wake in an incompressible fluid [12]. Interruptions in the flow are not considered. For the flow in the wake, Eqs. (2.2) and (2.3) take the form

$$
\begin{gathered}
U_\infty \frac{\partial \langle u\rangle}{\partial x} = -\frac{\partial}{\partial y}\langle u'v'\rangle,\\
U_\infty \frac{\partial \langle u'v'\rangle}{\partial x} + \langle v'^2\rangle \frac{\partial \langle u\rangle}{\partial y} = -\frac{\partial}{\partial y}\langle u'v'^2\rangle - \frac{2\nu_1+\nu_2}{\tau}\langle u'v'\rangle,\\
U_\infty \frac{\partial \langle v'^2\rangle}{\partial x} = -\frac{\partial \langle v'^3\rangle}{\partial y} - \frac{2\nu_1}{\tau}\langle v'^2\rangle - \frac{\nu_2}{\tau}\left(\langle v'^2\rangle - h\right),\\
U_\infty \frac{\partial \langle u'^2\rangle}{\partial x} = -\frac{\partial}{\partial y}\langle u'^2v'\rangle - \frac{2\nu_1}{\tau}\langle u'^2\rangle - \frac{\nu_2}{\tau}\left(\langle u'^2\rangle - h\right) - 2\langle u'v'\rangle\frac{\partial\langle u\rangle}{\partial y},\\
U_\infty \frac{\partial \langle w'^2\rangle}{\partial x} = -\frac{2\nu_1}{\tau}\langle w'^2\rangle - \frac{\nu_2}{\tau}\left(\langle w'^2\rangle - h\right) - \frac{\partial}{\partial y}\langle w'^2v'\rangle.
\end{gathered}
\tag{3.1}
$$

The equations for the third moments, closed by means of Eq. (2.6), have the form

$$
\begin{gathered}
U_\infty \frac{\partial\langle u'^2v'\rangle}{\partial x} + \langle v'^2\rangle\frac{\partial\langle u'^2\rangle}{\partial y} + 2\langle u'v'\rangle\frac{\partial\langle u'v'\rangle}{\partial y} + 2\langle u'v'^2\rangle\frac{\partial\langle u\rangle}{\partial y} = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle u'^2v'\rangle}{\tau},\\
U_\infty \frac{\partial\langle v'^3\rangle}{\partial x} + 3\langle v'^2\rangle\frac{\partial\langle v'^2\rangle}{\partial y} = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle v'^3\rangle}{\tau},\\
U_\infty \frac{\partial\langle w'^2v'\rangle}{\partial x} + \langle v'^2\rangle\frac{\partial\langle w'^2\rangle}{\partial y} = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle w'^2v'\rangle}{\tau},\\
U_\infty \frac{\partial\langle u'v'^2\rangle}{\partial x} + \langle u'v'\rangle\frac{\partial\langle v'^2\rangle}{\partial y} + 2\langle v'^2\rangle\frac{\partial\langle u'v'\rangle}{\partial y} + \langle v'^3\rangle\frac{\partial\langle u\rangle}{\partial y} = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle u'v'^2\rangle}{\tau}.
\end{gathered}
\tag{3.2}
$$

The equations for the third moments, closed by means of Eq. (2.5), are written in the form

$$
\begin{gathered}
U_\infty \frac{\partial\langle u'^2v'\rangle}{\partial x} + 2\langle u'v'^2\rangle\frac{\partial\langle u\rangle}{\partial y} - \langle u'^2\rangle\frac{\partial\langle v'^2\rangle}{\partial y} - 2\langle u'v'\rangle\frac{\partial\langle u'v'\rangle}{\partial y}\\
+\frac{\partial}{\partial y}h^2 + \frac{\partial}{\partial y}h\left(\langle u'^2\rangle - h\right) + \frac{\partial}{\partial y}h\left(\langle v'^2\rangle - h\right) = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle u'^2v'\rangle}{\tau},\\
U_\infty \frac{\partial\langle v'^3\rangle}{\partial x} - 3\langle v'^2\rangle\frac{\partial\langle v'^2\rangle}{\partial y} + 3\frac{\partial h^2}{\partial y} + 6\frac{\partial}{\partial y}h\left(\langle v'^2\rangle - h\right) = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle v'^3\rangle}{\tau},\\
U_\infty \frac{\partial}{\partial x}\langle w'^2v'\rangle - \langle w'^2\rangle\frac{\partial\langle v'^2\rangle}{\partial y} + \frac{\partial h^2}{\partial y} + \frac{\partial}{\partial y}h\left(\langle w'^2\rangle - h\right) + \frac{\partial}{\partial y}h\left(\langle v'^2\rangle - h\right) = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle u'^2v'\rangle}{\tau},\\
U_\infty \frac{\partial}{\partial x}\langle u'v'^2\rangle + \langle v'^3\rangle\frac{\partial\langle u\rangle}{\partial y} - \langle v'^2\rangle\frac{\partial\langle u'v'\rangle}{\partial y} - 2\langle u'v'\rangle\frac{\partial\langle v'^2\rangle}{\partial y} + 3\frac{\partial}{\partial y}h\langle u'v'\rangle = -\left(3\nu_1+\frac{3\nu_2}{2}\right)\frac{\langle u'v'^2\rangle}{\tau}.
\end{gathered}
\tag{3.3}
$$

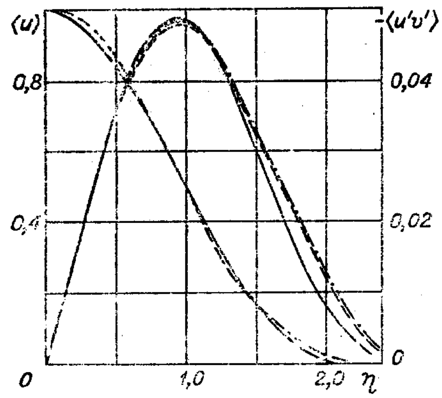

Fig. 2

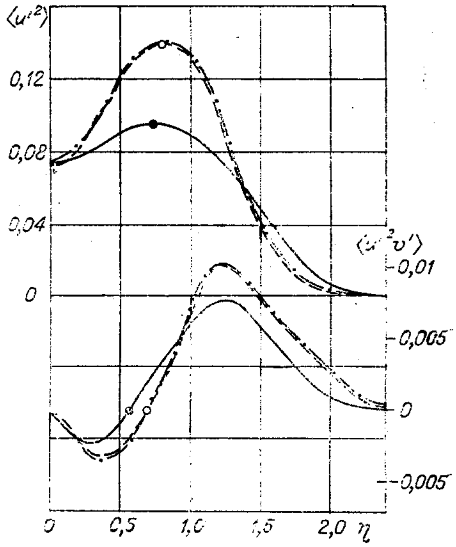

Fig. 3

Thus, in both cases of closure we obtain a system of nine differential equations for nine unknown functions. For convenience in further calculations, this system will henceforth be written in vector form (identical for both methods of closure)

$$\frac{\partial \mathbf{u}}{\partial x} + A(\mathbf{u}) \frac{\partial \mathbf{u}}{\partial y} = \mathbf{f}(\mathbf{u}), \tag{3.4}$$

where $\mathbf{u} = ||\langle u\rangle, \langle u'v'\rangle, \langle u'^2\rangle, \langle v'^2\rangle, \langle w'^2\rangle, \langle u'^2v'\rangle, \langle v'^3\rangle, \langle w'^2v'\rangle, \langle u'v'^2\rangle||$ is the unknown nine-component vector; $A(\mathbf{u})$ is a ninth-order square asymmetrical matrix; $\mathbf{f}(\mathbf{u})$ is the right side, algebraically nonlinear relative to the unknown vector. Matrix $A(\mathbf{u})$ and vector $\mathbf{f}(\mathbf{u})$ may be constructed in an obvious way from systems (3.1), (3.2) and (3.1), (3.3) so that their specific form does not have to be shown here.

In both cases, closure by the method of thirteen moments (system (3.1), (3.3)), and closure using the hypothesis of quasinormality (system (3.1), (3.2)), Eq. (3.4) is of the hyperbolic type: it is associated with actual multiple characteristics. In the case of closure using the hypothesis of quasinormality, Eq. (3.4) has the characteristics: λ_1 ($\equiv dx/dy$) = $1/\sqrt{\langle v'^2\rangle}$ (quadruple), $\lambda_2 = 1/\sqrt{3\langle v'^2\rangle}$ (double), and three actual characteristics determined by cubic equation $\lambda^3 + (3\langle v'^2\rangle/\langle v'^3\rangle)\lambda^2 - (1/\langle v'^3\rangle) = 0$. (In the case of closure by the thirteen-moments method, the characteristic equation for (3.4) has a complicated form, and the calculation of all characteristics in explicit form is difficult).

Due to the symmetry of the wake, we need only examine one of its halves. On the symmetry axis (at y = 0 (see Fig. 1)), the boundary conditions have the form

$$\frac{\partial\langle u\rangle}{\partial y} = 0, \quad \langle u'v'\rangle = 0, \quad \frac{\partial\langle u_i'^2\rangle}{\partial y} = 0 \quad (i = 1, 2, 3), \quad \partial\langle u'v'^2\rangle/\partial y = 0,$$
$$\langle u_i'^2 v'\rangle = 0.$$

At $y \to \infty$, the unknown vector $\mathbf{u}$ should satisfy the homogeneous boundary condition.

To solve Eq. (3.4) (with both methods of closure), we used the method of finite differences. We used an implicit difference scheme of the "predictor—corrector" type [18, 19]. The "corrector" (explicit recalculation) in this scheme is realized in the form of a "law of conservation" for Eq. (3.4) within the limits of the difference cells of the calculating grid. The difference analog of Eq. (3.4) at the "predictor" step has the form

$$A_n\mathbf{u}_{n+1} + \mathrm{B}_n\mathbf{u}_n + C_n\mathbf{u}_{n-1} = \mathbf{D}_n, \tag{3.5}$$

where A, C, and C are ninth-order square matrices; **D** is a nine-component vector. To convert the matrix in solving Eq. (3.5) by the method of matrix playing, a technique is used that is more economical than transformation with the selection of a principal element. The technique is based on the obvious fact that, by virtue of the physical postulation of the original problem itself for Eq. (3.4), the transformed matrix should not be poorly defined (the coef-

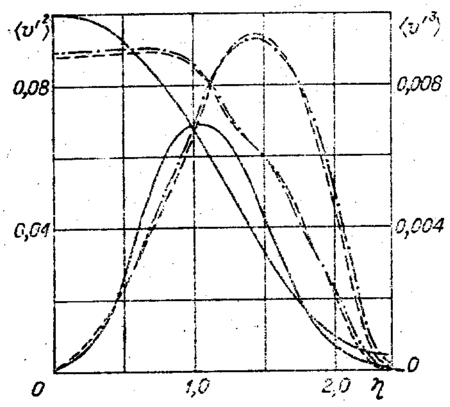

Fig. 4

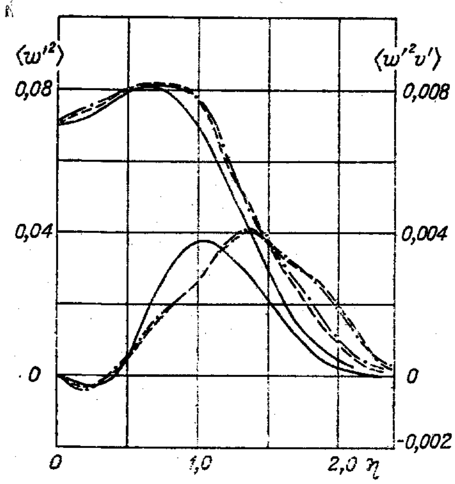

Fig. 5

ficients of the difference operator should not be confluent over the entire range of integration). The matrix therefore need not be transformed, and instead we can select its main element each time. The matrix can be transformed sequentially, by columns. As noted in [8], in the case of transforming a fourth-order matrix using the "by-column" method, the computing time(in the game algorithm) can be reduced by roughly 1.5 times compared to standard procedures presently available for transforming matrices with the selection of a main element. Here, as numerical experiments with fourth-order matrices have shown, four significant digits of the inverse matrix prove identical in both methods of transformation. The difference scheme used in this case is implicit, and the fact of its absolute computational stability has been confirmed by experiment (the performing of calculations with different ratios of difference-grid spacings).

The expression for the scale of turbulence was chosen on the basis of considerations of the number of dimensions and the similarity with an empirically observed flow structure [12],

$$L = k\delta,$$

where $k \simeq 0.3$ is an empirical constant; δ is the conditional width the wake — the distance between points at which the speed is half of the maximum.

4. Equation (3.5) was solved on a rectangular grid with spacings Δx and Δy in the direction of axes x and y (see Fig. 1) and the addition of grid elements during the calculation as a result of expansion of the wake, with an increase in the x coordinates. The size of the spacings was chosen so as to maintain the accuracy of the solution.

The numerical results are shown in Figs. 2-6. All functions have been normalized by means of the magnitude of the velocity defect on the wake axis. Dimensionless coordinate η represents the ratio of lateral coordinate y to the conditional width of the wake δ. Townsend's empirical points [12] are shown by the solid line in Figs. 2-6, while the dashed line shows the numerical results with closure by the hypothesis of fourth moments and the dot-and-dash line shows the results with closure by the method of thirteen moments. All curves pertain to the section of the wake $x/d \simeq 525$, where the second and third moments reach approximately the same states (over a distance of 50 spacings, from $x/d \simeq 475$ to $x/d \simeq 525$, the second moments in the numerical solution changed within 1%, while the third moments changed within 3%). Figure 3 shows the longitudinal intensity of the turbulence $\langle u'^2\rangle$ and the diffusion of this intensity in the transverse direction $\langle u'^2v'\rangle$. First of all, the results of the calculations are similar using both methods of closure, but the point on axis η where the numerical solution for the third moment $\langle u'^2v'\rangle$ vanishes does not coincide with the point on the axis η where $\langle u'^2\rangle$ assumes a maximum value. Secondly, although these points are displaced in the numerical solution to the right (toward the outer boundary of the wake) compared to similar empirical points, their mutual shift is similar to that seen experimentally. This provides a basis for concluding that the phenomenon of apparent "negative" viscosity relative to the process of diffusion of the intensity of the turbulence, observable experimentally in the axial zone of a flat wake, may be described on the basis of a model of transport in which the third moments are the unknown functions and are determined from corresponding differential equations of conservation. It may be noted that numerical results for the second moments of

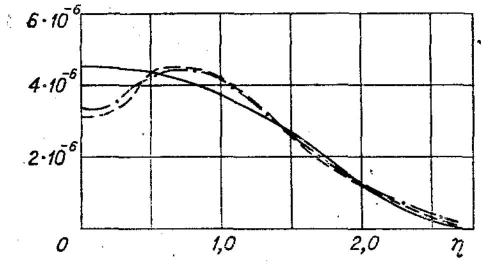

Fig. 6

velocity field pulsation in a flat wake were obtained earlier in [5] on the basis of the transport model discussed in Part 1. Here (in Part 1) it was assumed that any third moment can be expressed in the form of a gradient of second moments (closure hypothesis). However, in the numerical results shown in [5] there are no data on third moments or their comparison with empirical data. Figures 4-6 show the results of the numerical solution and its comparison with empirical points for the lateral intensity of the turbulence $\langle v'^2 \rangle$, its third moment (Fig. 4) for the intensity of turbulence $\langle w'^2 \rangle$ in the direction from the z axis (see Fig. 1), and its third moment (Fig. 5). Figure 6 shows the results for rate of turbulence energy dissipation $\varepsilon = 2\nu_1 E/\tau$. It is apparent that the peak of the dissipation curve of the numerical solution is shifted from the axis. This is evidently connected with the excessively isotropic nature of the expression for this quantity, failing to account for the anisotropic character of flow close to the axis. From the hypothesis of self-similarity, it follows that in using only one value of scale $L(L = L(x))$ and velocity $U_0(U_0 = U_0(x))$, systems of equations (3.1), (3.2) and (3.1), (3.3) permit a self-similar solution. It follows then from the corresponding conditions of self-similarity that the velocity defect on the wake axis U_0 and the conditional width of the wake $\delta(=L/k)$ change with the distance x downflow thus

$$U_0/U_\infty = c_0/\sqrt{x/d},\ \delta/d = (c_1/k)\sqrt{x/d}. \tag{4.1}$$

The empirical data in [12] provides evidence of the validity of self-similar expressions (4.1) for distances $x/d > 100$. The constant coefficients in Eqs. (4.1), found on the basis of this empirical data, are equal to: $c_0 \simeq 0.93$, $(c_1/k) \simeq 0.19$. In the numerical calculation, we obtained the same laws (4.1) of change in U_0 and δ with distance x, with the numerical coefficients having been equal to: $c_0 \simeq 1$, $c_1 \simeq 0.06$.

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FORMATION OF SUPERSONIC MOLECULAR BEAMS BY MEANS OF A SKIMMER

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One of the main problems in forming a molecular beam from a supersonic stream of low-density gas by the method in [1] is eliminating the distortions that arise from the interaction of the (forward) flowing stream with the skimmer.

Most investigations (see surveys [2, 3]) have been devoted to studying distortions of the intensity (density) of the molecular beam, while at the same time there has been almost no study in the literature of the effect of the skimmer interaction on the molecular velocity distribution function or its normalized moments (velocity of the flow and the forward-moving temperature). This reflects the narrow focus of the above studies, the principal purpose of which was to obtain molecular beams with extreme parameters: maximum intensity and minimum divergence [4].

One important recent trend is the investigation of relaxation processes in supersonic streams using a molecular beam [5, 6]. Thus, the study of mechanisms leading to distortions of velocity distribution functions and the search for conditions under which such distortion will not occur are prerequisite to expanding the scope of such investigations.

The present work is devoted to analysis and generalization of the results of experimental studies of the effect of skimmer interaction on the velocity distribution function conducted by the authors on a low-density gasdynamic tube at the Institute of Thermophysics of the Siberian Department of the Soviet Academy of Sciences equipped with a molecular-beam system [7]. The system has an apparatus providing for measurement of both parallel $T_{\parallel}$ (time-of-flight method of [8]) and perpendicular $T_{\perp}$ (electron-beam method of [9]) temperature. In all of the experiments, the working gas was commercially pure nitrogen.

1. The aim of the first series of measurements was to find the conditions under which the forward-moving temperature T would not be distorted by the interaction of the flowing stream with the skimmer. We made measurements of transverse profiles of the density of the molecular beam with a wide range of nozzle-skimmer distances x/d_*, stagnation pressure p_0, and the diameters of the inlet cross section of the skimmer d_s. In this work, the stagnation temperature was $T_0 = 293°K$, the diameter of the nozzle throat d_* was 2.11 mm. The design and dimensions of the skimmers used are detailed in [7]. We used the transverse profiles to determine density on the axis of the molecular beam n_b and the velocity relation $S_{\perp}$. The perpendicular temperature $T_{\perp}$ was computed from $S_{\perp}$ on the assumption that the hydrodynamic velocity was equal to the limiting velocity of the flow for known stagnation conditions.

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